

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS

4726 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4726
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

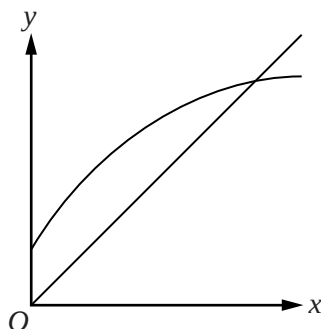
- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 Express $\operatorname{sech} 2x$ in terms of exponentials and hence, by using the substitution $u = e^{2x}$, find $\int \operatorname{sech} 2x \, dx$. [5]
- 2 A curve has polar equation $r = \cos \theta \sin 2\theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$. Find
- (i) the equations of the tangents at the pole, [2]
 - (ii) the maximum value of r , [4]
 - (iii) a cartesian equation of the curve, in a form not involving fractions. [3]
- 3 (i) By quoting results given in the List of Formulae (MF1), prove that $\tanh 2x \equiv \frac{2 \tanh x}{1 + \tanh^2 x}$. [2]
- (ii) Solve the equation $5 \tanh 2x = 1 + 6 \tanh x$, giving your answers in logarithmic form. [6]
- 4 It is given that the equation $x^4 - 2x - 1 = 0$ has only one positive root, α , and $1.3 < \alpha < 1.5$.

(i)



The diagram shows a sketch of $y = x$ and $y = \sqrt[4]{2x+1}$ for $x \geq 0$. Use the iteration $x_{n+1} = \sqrt[4]{2x_n+1}$ with $x_1 = 1.35$ to find x_2 and x_3 , correct to 4 decimal places. On the copy of the diagram show how the iteration converges to α . [3]

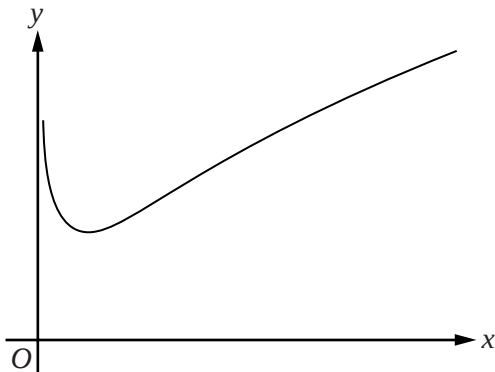
- (ii) For the same equation, the iteration $x_{n+1} = \frac{1}{2}(x_n^4 - 1)$ with $x_1 = 1.35$ gives $x_2 = 1.1608$ and $x_3 = 0.4077$, correct to 4 decimal places. Draw a sketch of $y = x$ and $y = \frac{1}{2}(x^4 - 1)$ for $x \geq 0$, and show how this iteration does not converge to α . [2]

- (iii) Find the positive root of the equation $x^4 - 2x - 1 = 0$ by using the Newton-Raphson method with $x_1 = 1.35$, giving the root correct to 4 decimal places. [4]

- 5 A function is defined by $f(x) = \sinh^{-1} x + \sinh^{-1} \left(\frac{1}{x} \right)$, for $x \neq 0$.

(i) When $x > 0$, show that the value of $f(x)$ for which $f'(x) = 0$ is $2 \ln(1 + \sqrt{2})$. [5]

(ii)



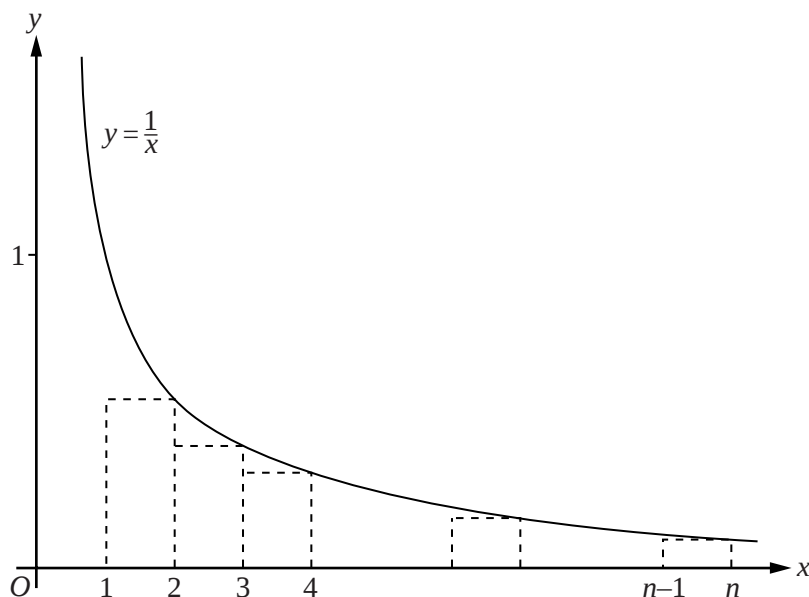
The diagram shows the graph of $y = f(x)$ for $x > 0$. Sketch the graph of $y = f(x)$ for $x < 0$ and state the range of values that $f(x)$ can take for $x \neq 0$. [3]

- 6 It is given that, for non-negative integers n ,

$$I_n = \int_0^\pi x^n \sin x \, dx.$$

(i) Prove that, for $n \geq 2$, $I_n = \pi^n - n(n-1)I_{n-2}$. [5]

(ii) Find I_5 in terms of π . [4]



The diagram shows the curve $y = \frac{1}{x}$ for $x > 0$ and a set of $(n - 1)$ rectangles of unit width below the curve. These rectangles can be used to obtain an inequality of the form

$$\frac{1}{a} + \frac{1}{a+1} + \frac{1}{a+2} + \dots + \frac{1}{b} < \int_1^n \frac{1}{x} dx.$$

Another set of rectangles can be used similarly to obtain

$$\int_1^n \frac{1}{x} dx < \frac{1}{c} + \frac{1}{c+1} + \frac{1}{c+2} + \dots + \frac{1}{d}.$$

- (i) Write down the values of the constants a and c , and express b and d in terms of n . [3]

The function f is defined by $f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln n$, for positive integers n .

- (ii) Use your answers to part (i) to obtain upper and lower bounds for $f(n)$. [4]

- (iii) By using the first 2 terms of the Maclaurin series for $\ln(1+x)$ show that, for large n ,

$$f(n+1) - f(n) \approx -\frac{n-1}{2n^2(n+1)}. \quad [5]$$

- 8** The curve C_1 has equation $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials of degree 2 and 1 respectively. The asymptotes of the curve are $x = -2$ and $y = \frac{1}{2}x + 1$, and the curve passes through the point $(-1, \frac{17}{2})$.

(i) Express the equation of C_1 in the form $y = \frac{p(x)}{q(x)}$. **[4]**

(ii) For the curve C_1 , find the range of values that y can take. **[4]**

Another curve, C_2 , has equation $y^2 = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are the polynomials found in part **(i)**.

(iii) It is given that C_2 intersects the line $y = \frac{1}{2}x + 1$ exactly once. Find the coordinates of the point of intersection. **[4]**

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THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.



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