

Monday 22 June 2015 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 By first expressing $\tanh y$ in terms of exponentials, prove that $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. [3]
- 2 It is given that $f(x) = \ln(1 + \sin x)$. Using standard series, find the Maclaurin series for $f(x)$ up to and including the term in x^3 . [4]
- 3 By first completing the square, find the exact value of $\int_{\frac{1}{2}}^1 \frac{1}{\sqrt{2x-x^2}} dx$. [5]
- 4 It is given that $I_n = \int_0^1 x^n e^{-x} dx$ for $n \geq 0$.
- (i) Show that $I_n = nI_{n-1} + k$ for $n \geq 1$, where k is a constant to be determined. [3]
- (ii) Find the exact value of I_3 . [3]
- (iii) Find the exact value of $990I_8 - I_{11}$. [3]
- 5 It is given that $y = \sin^{-1}2x$.
- (i) Using the derivative of $\sin^{-1}x$ given in the List of Formulae (MF1), find $\frac{dy}{dx}$. [1]
- (ii) Show that $(1 - 4x^2)\frac{d^2y}{dx^2} = 4x\frac{dy}{dx}$. [3]
- (iii) Hence show that $(1 - 4x^2)\frac{d^3y}{dx^3} - 12x\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = 0$. [2]
- (iv) Using your results from parts (i), (ii) and (iii), find the Maclaurin series for $\sin^{-1}2x$ up to and including the term in x^3 . [3]

6 It is given that the equation $3x^3 + 5x^2 - x - 1 = 0$ has three roots, one of which is positive.

(i) Show that the Newton-Raphson iterative formula for finding this root can be written

$$x_{n+1} = \frac{6x_n^3 + 5x_n^2 + 1}{9x_n^2 + 10x_n - 1}. \quad [3]$$

(ii) A sequence of iterates x_1, x_2, x_3, \dots which will find the positive root is such that the magnitude of the error in x_2 is greater than the magnitude of the error in x_1 . On the graph given in the Printed Answer Book, mark a possible position for x_1 . [1]

(iii) Apply the iterative formula in part (i) when the initial value is $x_1 = -1$. Describe the behaviour of the iterative sequence, illustrating your answer on the graph given in the Printed Answer Book. [2]

(iv) A sequence of approximations to the positive root is given by x_1, x_2, x_3, \dots . Successive differences

$x_r - x_{r-1} = d_r$, where $r \geq 2$, are such that $d_r \approx k(d_{r-1})^2$ where k is a constant.

Show that $d_4 \approx \frac{d_3^3}{d_2^2}$ and demonstrate this numerically when $x_1 = 1$. [4]

(v) Find the value of the positive root correct to 5 decimal places. [2]

7 It is given that $f(x) = \frac{x^2 - 25}{(x-1)(x+2)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Write down the equations of the asymptotes of the curve $y = f(x)$. [2]

(iii) Find the value of x where the graph of $y = f(x)$ cuts the horizontal asymptote. [2]

(iv) Sketch the graph of $y^2 = f(x)$. [2]

8 It is given that $f(x) = 2 \sinh x + 3 \cosh x$.

(i) Show that the curve $y = f(x)$ has a stationary point at $x = -\frac{1}{2} \ln 5$ and find the value of y at this point. [4]

(ii) Solve the equation $f(x) = 5$, giving your answers exactly. [5]

Question 9 begins on page 4.

9 The equation of a curve in polar coordinates is $r = 2 \sin 3\theta$ for $0 \leq \theta \leq \frac{1}{3}\pi$.

(i) Sketch the curve. [2]

(ii) Find the area of the region enclosed by this curve. [4]

(iii) By expressing $\sin 3\theta$ in terms of $\sin \theta$, show that a cartesian equation for the curve is

$$(x^2 + y^2)^2 = 6x^2y - 2y^3. \quad [5]$$

END OF QUESTION PAPER

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