

**GCE**

**Mathematics**

Unit **4725**: Further Pure Mathematics 1

Advanced Subsidiary GCE

**Mark Scheme for June 2016**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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## 1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

**Subject-specific Marking Instructions for GCE Mathematics Pure strand**

- a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

## g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance
1.	$\frac{1}{2}n(n+1)(2n+1) - n(n+1) - n$ $\frac{1}{2}n(2n+3)(n-1) \text{ or } n(n+\frac{3}{2})(n-1)$	M1* A1 A1  DM1 A1  <b>[5]</b>	Expand and attempt to use standard series, at least one used correctly Any two terms correct, may be unsimplified All terms correct  Attempt to find 3 factors Obtain correct answer
2	(i) $x = 2\sqrt{3} \cos(-\frac{\pi}{3}), y = 2\sqrt{3} \sin(-\frac{\pi}{3}), x^2 + y^2 = 12, y = -x\sqrt{3}$  $\sqrt{3} - 3i$	M1   A1  <b>[2]</b>	Correct trig expression for x or y allow a positive angle or 2 equations for x and y, not involving trig  Obtain correct answer as a complex number, extra answers not rejected gets A0
	(ii) $\sqrt{3} + 3i$  $-1 - (4\sqrt{3})i$ $-\frac{1}{49} + \frac{4\sqrt{3}}{49}i$	B1ft M1 A1 M1 A1  <b>[5]</b>	Correct conjugate seen, ft from their z in (i) Expand denominator Correct value seen Attempt to rationalise Obtain correct answer a.e.e.f. <b>N.B.</b> if 2 answers given in (i) award marks for better solution in (ii)

Question	Answer	Marks	Guidance
3. (i)	$\alpha + \beta = -\frac{1}{k}, \alpha\beta = 1$	B1 [1]	State correct values
(ii)	<p>Either</p> $\alpha\beta + \frac{1}{\alpha\beta} + \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$ <p>Or</p> $(\alpha + \beta)(\beta + \alpha)$ $\frac{1}{k^2} \text{ or } k^{-2}$	M1 M1 A1  M2 A1  M1 A1 [5]	Expand expression Use correct process for $\alpha^2 + \beta^2$ Obtain correct expression  State $\alpha = \frac{1}{\beta}$ and $\beta = \frac{1}{\alpha}$ and substitute into given expression Obtain correct expression  Use their value(s) in their expression Obtain correct single term answer
4. (i)	$(5a - 3b \ 10 \ 0)$	B1 B1 [2]	2 elements correct, must be a $1 \times 3$ matrix (not coordinates) 3 <sup>rd</sup> element correct
(ii)	$(6b - 5)$	M1 A1 [2]	Single value Correct answer, must be a matrix
(iii)	$\begin{pmatrix} 6a & 12 & 18 \\ 4a & 8 & 12 \\ -a & -2 & -3 \end{pmatrix}$	M1 A1 [2]	Obtain a $3 \times 3$ matrix All elements correct
5.	$3(2 \times 3^n - 1) + 2$	B1 M1 A1 B1 [4]	Show clearly result true for $n = 1$ , accept $(u_1) = 2 \times 3 - 1 = 5$ Substitute for $u_n$ in recurrence relation Establish correct result for $u_{n+1}$ convincingly Clear statement of induction conclusion, provided 1 <sup>st</sup> 3 marks earned



Question		Answer	Marks	Guidance
6.	(i)	$3 + 3i$ $\sqrt{5}$ $ z - 3 - 3i  = \sqrt{5}$	B1 B1 M1 A1ft <b>[4]</b>	Obtain centre as a complex number, allow (3, 3) but not (3, 3i), give if correct in locus equation Obtain correct radius a.e.f. condone decimals Use correct form for locus Obtain correct answer from their centre and radius, $z = x + iy$ is acceptable
	(ii)	Circle, centre in 1st quadrant, not touching or intersecting axes Straight line with –ve slope through (3, 3)	B1 B1 <b>[2]</b>	Centre need not be (3, 3) (3, 3) may be implied by working from (i) must be longer than the diameter. N.B. No circle drawn, B1 for $l$ may be earned
	(iii)	$2 + 5i$ $4 + i$	M1 A1 A1 <b>[3]</b>	Gradient of $l$ is $-2$ used or attempt to solve Cartesian equations for $C$ and $l$ M0 if equations for $C$ and/or $l$ contain $i$ . Obtain correct answers, must be complex numbers

Question	Answer	Marks	Guidance
7.	(i) Shear, $x$ -axis invariant and e.g. $(0, 1) \rightarrow (3, 1)$	B1 B1 [2]	Must be a shear, otherwise 0/2. For invariant only allow parallel to or along $x$ -axis, in $x$ direction, for image allow $0.322^\circ$ , $18.4^\circ$ , $\tan^{-1}(1/3)$ or the complement, ignore scale factor if all OK otherwise. Column vectors for coordinates OK
	(ii) <i>Either</i> $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -3 & -1 \\ -1 & 0 \end{pmatrix} \text{ or } \mathbf{PQ} = \mathbf{M}$ $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ <p style="text-align: center;">reflection in <math>y = -x</math></p> <p><i>Or</i></p> $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1  M1 A2  B2 [6]  B2 B2 B2	Obtain correct matrix equation  Sensible attempt at multiplication of a pair of $2 \times 2$ matrices Obtain correct answer, A1 for 3 elements correct  Must be describing Q If they say $M = QP \rightarrow P^{-1}M = Q$ , they can only get (possibly) M1 B2 if they obtain “correct” matrix for Q  Diagram showing at least unit square and image under $\mathbf{M}$ coordinates shown State reflection in $y = -x$ Correct matrix

Question	Answer	Marks	Guidance
8.	(i) $\frac{2r+3-(2r+1)}{(2r+1)(2r+3)}$ or $\frac{2r+3-2r-1}{(2r+1)(2r+3)} = \frac{2}{(2r+1)(2r+3)}$	B1 [1]	Establish <b>given</b> result correctly, might be a “0 = 0” type verification, or partial fractions
	(ii)  1/3 – 1/5 and 1/5 – 1/7 1/(2n + 1) – 1/(2n + 3)  $\frac{n}{3(2n + 3)}$	M1 M1 A1 A1 M1 A1 [6]	Express terms as differences using (i) Attempt this for at least 1 <sup>st</sup> two and last term First two terms correct, do not penalise missing factor of 2 Last term correct, do not penalise missing factor of 2, allow in terms of e.g. r or k Show correct cancelling Obtain a correct single fraction (denominator could be expanded) must be in terms of n. N.B. Be on look out for f(r) – f(r + 1) approach
	(iii)  <i>Either</i> $\frac{1}{6} - \frac{n-1}{3(2n+1)}$  $\frac{1}{2(2n+1)}$  <i>Or</i>  $\frac{1}{(2n+1)}$ $\frac{1}{2(2n+1)}$	M1 A1  A1 [3]  M1 A1 A1	Use sum to $\infty$ – sum to $n-1$ or $n$ Obtain correct unsimplified expression, do not penalise missing factor of 2  Obtain a correct single fraction ( denominator could be expanded)  Start differences at $r = n$ Obtain correct remaining term Obtain a correct single fraction (denominator could be expanded)

Question	Answer	Marks	Guidance
9. (i)	$a^2 - 8a + 15$	M1 M1 A1  [3]	Show correct expansion process for $3 \times 3$ condone sign errors Correct expansion for any $2 \times 2$ , may use Cramer's rule which is M2 Obtain <b>given</b> answer correctly
	(ii) $\det \mathbf{X} = 3^2 - 3 \times 8 + 15 = 0$ e.g. $3y + 5z = 5$ , or $3x - 7z = -4$ , or $5x + 7y = 5$	B1 M1 A1  [3]	And must state not a unique solution or equivalent Attempt to solve equations Find a repeated equation and state consistency N.B. They may solve the equations first then deduce consistent and non-unique and gets 3/3 (possibly)
10. (i)	$x^2 - y^2 = 9$ , $xy = 20$  $\pm(5 + 4i)$	M1 A1 M1 M1 A1 A1  [6]	Equate real and imaginary parts of $9 + 40i$ and $(x + iy)^2$ Obtain correct equations a.e.f., e.g. allow $2ixy = 40i$ Rearrange to obtain quadratic in $x^2$ or $y^2$ Solve quadratic in $x^2$ or $y^2$ and square root Obtain one correct answer as a complex number Obtain other answer as complex number a.e.f. $\pm 5 \pm 4i$ is OK for A2, $\pm(5 \pm 4i)$ scores A1 only
	(ii)	B1  [1]	Use sum and product of roots or solve equation or substitute into equation
	(iii) $u = \frac{1}{5 + 4i}$ $\pm(\frac{5}{41} + \frac{4}{41}i)$ , $\pm(\frac{5}{41} - \frac{4}{41}i)$	B1 M1  A1 A1  [4]	Use substitution to obtain one of the <b>equations</b> from other <b>equation</b> , (must have = 0) Use result from (i) ( either root )  Obtain any 2 correct answers Obtain other 2 correct answers  <b>S.C.:</b> If they solve quadratic for $u^2$ directly, use scheme above, M1 when they have used (i) to get a value for $u$ .

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