OCR
Oxford Cambridge and RSA

GCE

Mathematics

Unit **4726**: Further Pure Mathematics 2

Advanced GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

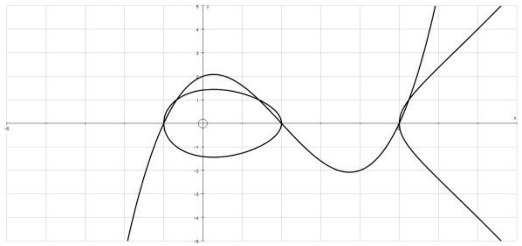
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

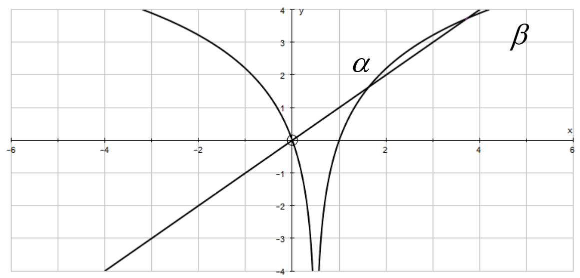
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Mark	Guidance	
1	(i)		$\left(e^x + e^{-x}\right)^3 = e^{3x} + 3e^x + 3e^{-x} + e^{-3x}$	M1	Doing the expansion	
			$= \left(e^{3x} + e^{-3x}\right) + 3\left(e^x + e^{-x}\right)$	A1		
			$\Rightarrow \left(2\cosh x\right)^3 = 2\cosh 3x + 6\cosh x$	M1	Relating cosh3x to exponentials correctly	
			$\Rightarrow 8\cosh^3 x = 2\cosh 3x + 6\cosh x$	A1		
			$\Rightarrow \cosh 3x = 4\cosh^3 x - 3\cosh x$	[4]		
	(ii)		$\Rightarrow \cosh 3x = 4\cosh^3 x - 3\cosh x = 6\cosh x$	M1	Using result of (i)	
			$\Rightarrow 4\cosh^3 x = 9\cosh x$			
			$\Rightarrow \cosh^2 x = \frac{9}{4}$ since $\cosh x \neq 0$	A1	At least one rejection needs to be stated. Or cosh$x \geq 1$	
			$\Rightarrow \cosh x = (\pm)\frac{3}{2}$ $\cosh x \neq -\frac{3}{2}$	A1		
			$\Rightarrow x = \pm\ln\left(\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 - 1}\right)$	A1 A1	A1 for each in exact form Deduct from 5 marks 1 mark for additional incorrect answers	
			$= \pm\ln\left(\frac{3}{2} + \frac{1}{2}\sqrt{5}\right)$ or $\ln\left(\frac{3}{2} \pm \frac{1}{2}\sqrt{5}\right)$	[5]		

Question			Answer	Mark	Guidance	
			Alternative: $\cosh 3x = 6\cosh x \Rightarrow \frac{1}{2}\left(e^{3x} + e^{-3x}\right) = 3\left(e^{x} + e^{-x}\right)$ $\Rightarrow e^{3x} - 6e^{x} - 6e^{-x} + e^{-3x} = 0 \Rightarrow e^{6x} - 6e^{4x} - 6e^{2x} + 1 = 0$ let $y = e^{2x}$ $\Rightarrow y^3 - 6y^2 - 6y + 1 = 0 \Rightarrow (y+1)\left(y^2 - 7y + 1\right) = 0$ $\Rightarrow y = -1, \frac{7 \pm \sqrt{45}}{2}$ $e^{2x} \neq -1 \Rightarrow e^{2x} = \frac{7 \pm \sqrt{45}}{2} \Rightarrow x = \frac{1}{2}\ln\left(\frac{7 \pm \sqrt{45}}{2}\right) = \frac{1}{2}\ln\left(\frac{7 \pm 3\sqrt{5}}{2}\right)$	**M1** **A1** **A1** **A1** **A1**	Using exponentials Cubic in factorised form Rejection of $y = -1$ must be stated. **oe** in exact form Deduct from 5 marks 1 mark for additional incorrect answers	

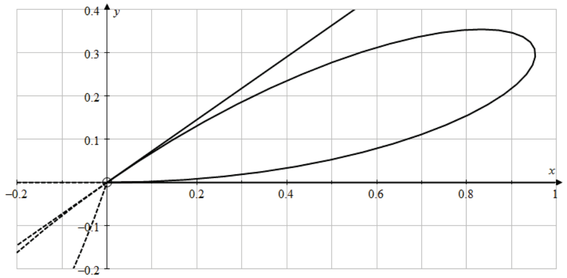
Question			Answer	Mark	Guidance	
2			$f(x)=\frac{x(x-1)}{(x+1)(x^2+1)}\equiv\frac{A}{(x+1)}+\frac{Bx+C}{(x^2+1)}$	M1	Correct partial fractions	
			$\Rightarrow A(x^2+1)+(Bx+C)(x+1)\equiv x(x-1)$ For e.g. equate coefficients $\Rightarrow A+B=1,\quad B+C=-1,\quad A+C=0$	M1	Dep on 1st M	Or sub values of x or division
			$\Rightarrow A=1, B=0, C=-1$ $\Rightarrow (f(x))=\frac{1}{(x+1)}-\frac{1}{(x^2+1)}$	A1	Dep on both M marks.	
			$\Rightarrow \int_0^1 f(x)dx=\int_0^1\left(\frac{1}{(x+1)}-\frac{1}{(x^2+1)}\right)dx$	B1	ft for integrating 1st term correctly (A/(x + 1) A ≠0)	
			$=\left[\ln(1+x)-\tan^{-1}x\right]_0^1=\ln 2-\frac{\pi}{4}$	B1	ft for subsequent term(s) correctly	
				B1	in exact form as dep on both previous B marks	
				[6]		

Question			Answer	Mark	Guidance	
3				B1	Symmetric but not for reflecting original curve	
				B1	Both ranges only and nothing more	
				B1	All parts cut *x*-axis at 90^0	
				B1	Both parts cut original curve at *y* = *k* and central part "egg shaped" *k* only approximately 1	Approximate consistency
			(-1, 0), (2, 0), (5, 0) $\left(0, \sqrt{2}\right), \left(0, -\sqrt{2}\right)$	B1	All 5points given!	
				[5]		

Question			Answer	Mark	Guidance	
4	(i)		$x = 0$ in equation satisfies as $e^0 = 1$.	B1 [1]		
	(ii)	(a)		B1 B1 B1 [3]	Asymptote between $x = 0$ and where it crosses x axis . +ve roots clear LH branch going through origin LH branch does not have to be complete	Allow one branch. SC1 $y = \left(\ln(2x-1)\right)^2$
		(b)	Staircase seen near middle root to be converging to β.	B1 [1]	Either starting point shown with vertical line from axis to curve or arrows on staircase lines	Follow through their curve where there are two positive roots
	(ii)	(c)	$x_1 = 3.75$ $x_2 = 3.743604...$ Leading to 3.733	B3 [3]	For correct answer B2 for 3.734 B1 for sight of 3.7436...	
	(iii)		$f(x) = (2x-1)^2 - e^x$ $\Rightarrow f'(x) = 4(2x-1) - e^x$ $\Rightarrow x_{r+1} = x_r - \dfrac{(2x-1)^2 - e^x}{4(2x-1) - e^x}$ $\Rightarrow x_2 = 1.629382......., \ x_3 = 1.629053$ Root = 1.6291	B1 M1 A1 A1 [4]	f'(x) correct **soi by x_2** Use of formula **soi by x_2** x_2 to 2dp or better Correct root stated to 5sf	 f(x) correct and their f'(x) At least 2 iterates shown

Question			Answer	Mark	Guidance	
5	(i)		$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{1+4x^2}$	B1	For first diffn	
			$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2\left(1+4x^2\right)^{-2}\times 8x=\frac{-16x}{\left(1+4x^2\right)^2}=-4x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	M1	Diffn again and making comparison	
				A1		
				[3]		
	(ii)		When $x=0,\ y=0, \frac{\mathrm{d}y}{\mathrm{d}x}=2,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$	B1	Soi by final answer	See below for alternative differentiation.
			$\Rightarrow \frac{\mathrm{d}^3y}{\mathrm{d}x^3}+4\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+8x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$	M1	Differentiate the equation given	SC4 Final formula from formula book including sight of $(2x)^3$.
			When $x=0,\ \frac{\mathrm{d}^3y}{\mathrm{d}x^3}+16=0$	A1	For – 16 www	SC2 if final form only seen (i.e. no working)
			$\Rightarrow (y)=2x-\frac{8x^3}{3}$	A1	Final answer	SC0 if final form only and wrong
				[4]		
	(iii)		$x=\frac{1}{2}\Rightarrow \tan^{-1}1=\frac{\pi}{4}$	B1	soi	
			In series $x=1-\frac{1}{3}=\frac{2}{3}$			
			$\Rightarrow$ Estimate for $\pi=\frac{8}{3}=2.666....$	B1	For showing to 1sf π = 3 which is correct but to 2sf π = 2.7 which is not. www	
			which, correct to 1sf, = 3	[2]		

Question			Answer	Mark	Guidance	
			Alternative differentiation: $y'' = \frac{-16x}{(1+4x^2)^2} \Rightarrow y''' = \frac{-16(1+4x^2)^2 + 256(1+4x^2)x^2}{(1+4x^2)^4}$ or $y'' = -16x(1+4x^2)^{-2} \Rightarrow y''' = 256x^2(1+4x^2)^{-3} - 16(1+4x^2)^{-2}$		M1 is for two terms on top M1 is for 2 terms	

Question			Answer	Mark	Guidance	
6	(i)			B1 B1 B1 B1 [4]	Single enclosed loop in 1st quadrant tangent at origin (not necessarily seen) less than $\theta = \frac{\pi}{4}$ $\theta = \frac{\pi}{5}$ stated $\theta = 0$ stated –1 any extra "tangents" if 2 are correct	
	(ii)		$\theta = \frac{\pi}{10}$, $x = r\cos\theta = \cos\frac{\pi}{10}$ (≈ 0.951) $y = r\sin\theta = \sin\frac{\pi}{10}$ (≈ 0.309)	B1 B1 [2]	For θ. Allow cartesian $y = \left(\tan\frac{\pi}{10}\right)x$ For both *x* and *y* . Allow decimal values to 3sf or better	
	(iii)		$A = \frac{1}{2}\int_0^{\pi/5} r^2 \mathrm{d}\theta = \frac{1}{2}\int_0^{\pi/5} \sin^2 5\theta \mathrm{d}\theta$ $= \frac{1}{4}\int_0^{\pi/5} (1-\cos 10\theta)\mathrm{d}\theta = \frac{1}{4}\left[\theta - \frac{1}{10}\sin 10\theta\right]_0^{\pi/5}$ $= \frac{1}{4}\left(\frac{\pi}{5} - \frac{1}{10}\sin 2\pi\right) = \frac{\pi}{20}$	M1 M1 A1 A1 [4]	Correct formula for area with correct limits Correct method to get integrand Dep on 2nd M Integral – ignore limits	or $A = \int_0^{\pi/10} r^2 \mathrm{d}\theta$

Question			Answer	Mark	Guidance	
7	(i)		Construct rectangles from $x = 1$ of height y to n or ∞ Sum of areas of rectangles $= \frac{1}{1} + \frac{1}{2} + ... = \sum_1^\infty \frac{1}{x}$ This area is bigger than the area under the curve $A = \int_1^\infty \frac{1}{x} dx = [\ln x]_1^\infty = \infty$ Since *Sum* > *A*, the sum is infinite.	M1 A1 B1 A1 [4]	Or sum from 1 to n soi from diagram For sum and inequality stated or implied by diagram Integral from 1 to n +1or ∞ Conclusion	Or to n - 1 Condone comparison of areas for 1 to n and 1 to n - 1 If n- 1 above then this integral to n
	(ii)		Construct rectangles from $x = 2$ to the left of height y to n or ∞ Sum of areas of rectangles $= \frac{1}{2^2} + \frac{1}{3^2} + ... = \sum_2^\infty \frac{1}{x^2}$ This area is less than the area under the curve $A = \int_1^\infty \frac{1}{x^2} dx = \left[-\frac{1}{x}\right]_1^\infty = 0 - -1 = 1$ Since *Sum* < *A* , $\frac{1}{2^2} + \frac{1}{3^2} + ... = \sum_2^\infty \frac{1}{x^2} < 1$ $\Rightarrow \sum_1^\infty \frac{1}{x^2} < 1 + 1$ Upper limit = 2	M1 A1 B1 M1 A1 [5]	Rectangles must be under curve For sum and inequality stated or implied by diagram Area under curve For adding 1 to both sides, may appear earlier	May include the rectangle from x= 1 to the left

Question		Answer	Mark	Guidance	
8	(i)	$I_n = \int_0^{\pi/4} \sec^n x \, dx$			
		$u = \sec^{n-2} x \qquad dv = \sec^2 x \, dx$ $du = (n-2)\sec^{n-3} x.\sec x \tan x \, dx \qquad v = \tan x$	M1	Attempt at integration by parts with correct *u* and *v*'	
		$\Rightarrow I_n = \left[\sec^{n-2} x \tan x\right]_0^{\pi/4} - \int_0^{\pi/4} (n-2)\sec^{n-3} x.\sec x \tan x.\tan x \, dx$	A1	"*uv*" term must be seen	
		$= \left(\sqrt{2}\right)^{n-2} - (n-2)\int_0^{\pi/4} \sec^{n-2} x.\tan^2 x.dx$	A1	Evaluating the first term	
		$= \left(\sqrt{2}\right)^{n-2} - (n-2)\int_0^{\pi/4} \sec^{n-2} x\left(\sec^2 x - 1\right).dx$	M1	Dep on 1st M Splitting integral using $\tan^2 x = \sec^2 x - 1$	
		$= \left(\sqrt{2}\right)^{n-2} - (n-2)\left(I_n - I_{n-2}\right) \Rightarrow (n-1)I_n = \left(\sqrt{2}\right)^{n-2} + (n-2)I_{n-2}$	A1 [5]		
	(ii)	$I_8 = \frac{1}{7}\left(2^3 + 6I_6\right) = \frac{1}{7}\left(2^3 + 6.\frac{1}{5}\left(2^2 + 4I_4\right)\right)$	B1	Correct statement of formula seen anywhere	
		$= \frac{1}{7}\left(2^3 + 6.\frac{1}{5}\left(2^2 + 4.\frac{1}{3}\left(2 + 2I_2\right)\right)\right)$			
		$= \frac{1}{7}\left(8 + \frac{6}{5}\left(4 + \frac{16}{3}\right)\right) = \frac{1}{7}\left(8 + \frac{6}{5}.\frac{28}{3}\right)$	B1	Substitution of I_2 seen oe	
		$= \frac{1}{7}.\frac{96}{5} = \frac{96}{35}$	B1	Conclusion	
		Alternative: $I_2 = 1 \qquad I_4 = \frac{4}{3} \qquad I_6 = \frac{28}{15} \Rightarrow I_8 = \frac{96}{35}$	[3]	B1 for I_2 B1 For I_4 and I_6 B1	

Question			Answer	Mark	Guidance	
	(iii)		$I_2 = 1$ (which is rational). (Therefore I_{2n} is rational for $n = 1$).	B1	Allow $n = 2$	
			Let I_{2k} be rational for some value of k Then $I_{2(k+1)} = \frac{1}{(2k+1)}\left(\sqrt{2}^{2k} + 2kI_{2k}\right) = \frac{1}{(2k+1)}\left(2^k + 2kI_{2k}\right)$	M1	For statement plus reduction formula and begin to look at the $\sqrt{2}$ term	Assume true for $n = k$
			Statement that this is rational and must include $\sqrt{2}^{2k} = 2^k$ is rational	A1		
			So if I_{2k} is rational then $I_{2(k+1)}$ is rational. But I_2 is rational so I_{2k} is rational for all $n = k$	A1	Reduction formula must be correct	
				[4]		

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