

OCR

Oxford Cambridge and RSA

Friday 20 May 2016 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

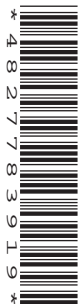
OCR supplied materials:

- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 Find $\sum_{r=1}^n (3r+1)(r-1)$, giving your answer in a fully factorised form. [5]
- 2 The complex number z has modulus $2\sqrt{3}$ and argument $-\frac{1}{3}\pi$. Giving your answers in the form $x+iy$, where x and y are exact real numbers, and showing clearly how you obtain them, find
- (i) z , [2]
- (ii) $\frac{1}{(z^* - 5i)^2}$. [5]
- 3 The quadratic equation $kx^2 + x + k = 0$ has roots α and β .
- (i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]
- (ii) Find the value of $\left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right)$ in terms of k . [5]
- 4 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} a & 2 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} b & 0 & 5 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$. Find
- (i) $5\mathbf{A} - 3\mathbf{B}$, [2]
- (ii) \mathbf{BC} , [2]
- (iii) \mathbf{CA} . [2]
- 5 The sequence u_1, u_2, u_3, \dots is defined by

$$u_1 = 5 \text{ and } u_{n+1} = 3u_n + 2 \text{ for } n \geq 1.$$
 Prove by induction that $u_n = 2 \times 3^n - 1$. [4]
- 6 In an Argand diagram the points A and B represent the complex numbers $5 + 4i$ and $1 + 2i$ respectively.
- (i) Given that A and B are the ends of a diameter of a circle C , find the equation of C in complex number form. [4]
- The perpendicular bisector of AB is denoted by l .
- (ii) Sketch C and l on a single Argand diagram. [2]
- (iii) Find the complex numbers represented by the points of intersection of C and l . [3]
- 7 The matrix $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ represents a transformation P .
- (i) Describe fully the transformation P . [2]
- The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} -3 & -1 \\ -1 & 0 \end{pmatrix}$.
- (ii) Given that \mathbf{M} represents transformation Q followed by transformation P , find the matrix that represents the transformation Q and describe fully the transformation Q . [6]

8 (i) Show that $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$. [1]

(ii) Hence find $\sum_{r=1}^n \frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. [6]

(iii) Find $\sum_{r=n}^{\infty} \frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. [3]

9 (i) The matrix \mathbf{X} is given by $\mathbf{X} = \begin{pmatrix} a & 3 & -2 \\ 0 & a & 5 \\ 1 & 2 & 1 \end{pmatrix}$. Show that the determinant of \mathbf{X} is $a^2 - 8a + 15$. [3]

(ii) Explain briefly why the equations

$$3x + 3y - 2z = 1$$

$$3y + 5z = 5$$

$$x + 2y + z = 2$$

do not have a unique solution and determine whether these equations are consistent or inconsistent. [3]

10 (i) Use an algebraic method to find the square roots of the complex number $9 + 40i$. [6]

(ii) Show that $9 + 40i$ is a root of the quadratic equation $z^2 - 18z + 1681 = 0$. [1]

(iii) By using the substitution $z = \frac{1}{u^2}$, find the roots of the equation $1681u^4 - 18u^2 + 1 = 0$. Give your answers in the form $x + iy$, where x and y are real. [4]

END OF QUESTION PAPER

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