

Monday 27 June 2016 – Morning

A2 GCE MATHEMATICS

4726/01 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4726/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

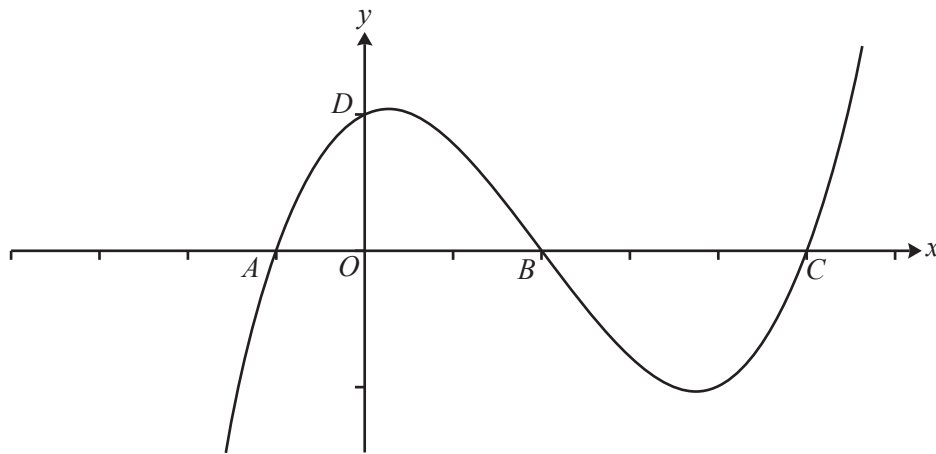
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

- 1 (i) By first expanding $(e^x + e^{-x})^3$, or otherwise, show that $\cosh 3x \equiv 4 \cosh^3 x - 3 \cosh x$. [4]
- (ii) Solve the equation $\cosh 3x = 6 \cosh x$, giving your answers in exact logarithmic form. [5]

- 2 It is given that $f(x) = \frac{x(x-1)}{(x+1)(x^2+1)}$. Express $f(x)$ in partial fractions and hence find the exact value of $\int_0^1 f(x) dx$. [6]

- 3 The diagram shows the curve $y = f(x)$. Points A , B , C and D on the curve have coordinates $(-1, 0)$, $(2, 0)$, $(5, 0)$ and $(0, 2)$ respectively.



On the copy of this diagram in the Printed Answer Book, sketch the curve $y^2 = f(x)$, giving the coordinates of the points where the curve crosses the axes. [5]

4 You are given the equation $(2x-1)^2 - e^x = 0$.

- (i) Verify that 0 is a root of the equation. [1]

There are also two other roots, α and β , where $0 < \alpha < \beta$.

- (ii) The iterative formula $x_{r+1} = \ln(2x_r - 1)^2$ is to be used to find a root of the equation.

- (a) Sketch the line $y = x$ and the curve $y = \ln(2x-1)^2$ on the same axes, showing the roots 0, α and β . [3]

- (b) By drawing a 'staircase' diagram on your sketch, starting with a value of x that is between α and β , show that this iteration does not converge to α . [1]

- (c) Using this iterative formula with $x_1 = 3.75$, find the value of β correct to 3 decimal places. [3]

- (iii) Using the Newton-Raphson method with $x_1 = 1.6$, find the root α of the equation $(2x-1)^2 - e^x = 0$ correct to 5 significant figures. Show the result of each iteration. [4]

5 It is given that $y = \tan^{-1} 2x$.

- (i) Find $\frac{dy}{dx}$ and show that $\frac{d^2y}{dx^2} + 4x \left(\frac{dy}{dx} \right)^2 = 0$. [3]

- (ii) Find the Maclaurin series for y up to and including the term in x^3 . Show all your working. [4]

- (iii) The result in part (ii), together with the value $x = \frac{1}{2}$, is used to find an estimate for π . Show that this estimate is only correct to 1 significant figure. [2]

- 6 The equation of a curve in polar coordinates is $r = \sin 5\theta$ for $0 \leq \theta \leq \frac{1}{5}\pi$.
- (i) Sketch the curve and write down the equations of the tangents at the pole. [4]
- (ii) The line of symmetry meets the curve at the pole and at one other point A . Find the equation of the line of symmetry and the cartesian coordinates of A . [2]
- (iii) Find the area of the region enclosed by this curve. [4]
- 7 (i) By using a set of rectangles of unit width to approximate an area under the curve $y = \frac{1}{x}$, show that $\sum_{x=1}^{\infty} \frac{1}{x}$ is infinite. [4]
- (ii) By using a set of rectangles of unit width to approximate an area under the curve $y = \frac{1}{x^2}$, find an upper limit for the series $\sum_{x=1}^{\infty} \frac{1}{x^2}$. [5]
- 8 It is given that $I_n = \int_0^{\frac{1}{4}\pi} \sec^n x \, dx$ where n is a positive integer.
- (i) By writing $\sec^n x = \sec^{n-2} x \sec^2 x$, or otherwise, show that
- $$(n-1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2} \text{ for } n > 1. \quad [5]$$
- (ii) Show that $I_8 = \frac{96}{35}$. [3]
- (iii) Prove by induction that I_{2n} is rational for all values of $n > 1$. [4]

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.