

# OCR

Oxford Cambridge and RSA

## Friday 17 June 2016 – Afternoon

### A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4727/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

1 In this question, give all non-real numbers in the form  $re^{i\theta}$  where  $r > 0$  and  $0 < \theta < 2\pi$ .

(i) Solve  $z^5 = 1$ . [2]

(ii) Hence, or otherwise, solve  $z^5 + 32 = 0$ . Sketch an Argand diagram showing the roots. [4]

2 Find the shortest distance between the lines  $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$  and  $\mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ . [4]

3 The differential equation

$$\frac{2}{y} - \frac{x}{y^2} \frac{dy}{dx} = \frac{1}{x^2}$$

is to be solved subject to the condition  $y = 1$  when  $x = 1$ .

(i) Show that  $y = \frac{1}{u}$  transforms the differential equation into

$$x \frac{du}{dx} + 2u = \frac{1}{x^2}. \quad [3]$$

(ii) Find  $y$  in terms of  $x$ . [7]

4 Let  $A$  be the set of non-zero integers.

(i) Show that  $A$  does not form a group under multiplication. [2]

(ii) State the largest subset of  $A$  which forms a group under multiplication. Show that this is a group. [3]

5 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 85 \cos x. \quad [8]$$

6 The planes  $\Pi_1$  and  $\Pi_2$  have equations

$$\mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 3 \quad \text{and} \quad \mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = 5$$

respectively. They intersect in the line  $l$ .

(i) Find cartesian equations of  $l$ . [4]

The plane  $\Pi_3$  has equation  $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} = 1$ .

(ii) Show that  $\Pi_3$  is parallel to  $l$  but does not contain it. [3]

(iii) Verify that  $(2, 0, 1)$  lies on planes  $\Pi_1$  and  $\Pi_3$ . Hence write down a vector equation of the line of intersection of these planes. [3]

- 7 (i) Use de Moivre's theorem to show that

$$\sin 6\theta \equiv \cos \theta (6 \sin \theta - 32 \sin^3 \theta + 32 \sin^5 \theta). \quad [5]$$

- (ii) Hence show that, for  $\sin 2\theta \neq 0$ ,

$$-1 \leq \frac{\sin 6\theta}{\sin 2\theta} < 3. \quad [7]$$

- 8 A non-commutative multiplicative group  $G$  of order eight has the elements

$$\{e, a, a^2, a^3, b, ab, a^2b, a^3b\},$$

where  $e$  is the identity and  $a^4 = b^2 = e$ .

- (i) Show that  $ba \neq a^n$  for any integer  $n$ . [2]
- (ii) Prove, by contradiction, that  $ba \neq a^2b$  and also that  $ba \neq ab$ . Deduce that  $ba = a^3b$ . [6]
- (iii) Prove that  $ba^2 = a^2b$ . [2]
- (iv) Construct group tables for the three subgroups of  $G$  of order four. [7]

**END OF QUESTION PAPER**

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