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RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

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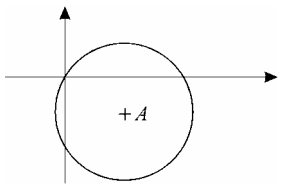
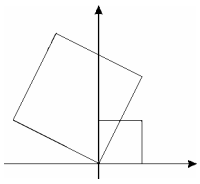
1	$\sum_{r=1}^{n} r(r+1) = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r = \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$	M1	For considering the two separate sums
		A1	For either correct sum formula stated
		A1	For completely correct expression
	$= \frac{1}{6}n(n+1)(2n+1+3) = \frac{1}{3}n(n+1)(n+2)$	M1	For factorising attempt
		A1 **5**	For showing given answer correctly
		5	
2	**(i)** $(p-q)+p+(p+q)=6 \Rightarrow p=2$	M1	For use of $\Sigma\alpha = -b/a$
		A1 **2**	For correct answer
	(ii) $2(2-q)(2+q)=-10$	B1√	For use of $\alpha\beta\gamma = -d/a$
	Hence $4-q^2=-5 \Rightarrow q=3$	M1	For expanding and solving for q^2
		A1 **3**	For correct answer
	(iii) *EITHER*: Roots are $-1, 2, 5$	B1√	For stating or using three numerical roots
	$-1\times2+2\times5+-1\times5=k$	M1	For use of $\Sigma\alpha\beta = c/a$
	i.e. $k=3$	A1√	For correct answer from their roots
	OR: Roots are $-1, 2, 5$	B1√	For stating or using three numerical roots
	Equation is $(x+1)(x-2)(x-5)=0$	M1	For stating and expanding factorised form
	Hence $k=3$	A1√ **3**	For correct answer from their roots
		8	
3	**(i)** $z^2=(2+\mathrm{i})^2=4+4\mathrm{i}+\mathrm{i}^2=3+4\mathrm{i}$	M1	For showing 3-term or 4-term expansion
		A1 **2**	For correct answer
	(ii) $4z-z^2=8+4\mathrm{i}-3-4\mathrm{i}=5$	B1	For correct value 5
	$zz^*=(2+\mathrm{i})(2-\mathrm{i})=5$	B1	For stating or using $z^*=2-\mathrm{i}$
		B1 **3**	For correct verification of given restult
	(iii) $\frac{z+1}{z-1}=\frac{3+\mathrm{i}}{1+\mathrm{i}}=\frac{(3+\mathrm{i})(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})}=\frac{4-2\mathrm{i}}{2}=2-\mathrm{i}$	B1	For correct initial form $\frac{3+\mathrm{i}}{1+\mathrm{i}}$
		M1	For multiplying top and bottom by $1-\mathrm{i}$
		A1 **3**	For correct answer $2-\mathrm{i}$
		8	
4	**(i)** $u_1=8$	B1 **1**	For correct value stated
	(ii) $3^{2(n+1)}-1-(3^{2n}-1)=9\times3^{2n}-3^{2n}=8\times3^{2n}$	B1	For stating or using $u_{n+1}=3^{2(n+1)}-1$
		M1	For relevant manipulation of indices in u_{n+1}
		A1 **3**	For showing given answer correctly
	(iii) u_1 is divisible by 8, from **(i)**	B1	For explicit check for u_1
	Suppose u_k is divisible by 8, i.e. $u_k=8a$	M1	For induction hypothesis u_k is mult. of 8
	Then $u_{k+1}=u_k+8\times3^{2k}=8(a+3^{2k})=8b$	M1	For obtaining and simplifying expr. for u_{k+1}
	i.e. u_{k+1} is also divisible by 8, and result follows by the induction principle	A1 **4**	For correct conclusion, stated and justified
		8	

5	**(i)**	$\text{LHS} = \dfrac{2r+1-(2r-1)}{(2r-1)(2r+1)} = \dfrac{2}{4r^2-1} = \text{RHS}$	M1	For correct process for adding fractions
			A1 **2**	For showing given result correctly
	(ii)	Sum is $\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\ldots+\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$	M1	For expressing terms as differences using **(i)**
			A1	For at least first two and last terms correct
		This is $1-\dfrac{1}{2n+1}$	M1	For cancelling pairs of terms
			A1 **4**	For any correct form
	(iii)	**(a)** Sum to infinity is 1	B1√ **1**	For correct value; follow their **(ii)** if cnvgt
		(b) Required sum is $\dfrac{1}{2n+1}$	B1√ **1**	For correct difference of their **(iii)(a)** and **(ii)**
			8	
6	**(i)**	(See diagram in part **(ii)** below)	B1	For point A correctly located
		$\lvert a\rvert = \sqrt{(3^2+4^2)} = 5$	B1	For correct value for the modulus
		$\arg a = -\tan^{-1}\left(\frac{3}{4}\right) = -0.644$	M1	For any correct relevant trig statement
			A1 **4**	For correct answer (radians or degrees)
	(ii)		B1	For any indication that locus is a circle
			B1	For any indication that the centre is at A
			B1 **3**	For a completely correct diagram
	(iii)	**(a)** $z = -6\text{i}$	B1 **1**	For correct answer
		(b) $z = 8-6\text{i}$	M1	For identification of end of diameter thru A
			A1 **2**	For correct answer
			10	
7	**(i)**	$\begin{pmatrix}1 & -2\\ 2 & 1\end{pmatrix}\begin{pmatrix}0 & 1 & 0 & 1\\ 0 & 0 & 1 & 1\end{pmatrix} = \begin{pmatrix}0 & 1 & -2 & -1\\ 0 & 2 & 1 & 3\end{pmatrix}$	M1	For at least one correct image
			A1	For all vertices correct
			A1 **3**	For correct diagram
	(ii)	The area scale-factor is 5	B1	For identifying det as area scale factor
		The transformed square has side of length $\sqrt{5}$	M1	For calculation method relating to large sq.
		So its area is 5 times that of the unit square	A1 **3**	For a complete explanataion
	(iii)	Angle is $\tan^{-1}(2) = 63.4°$	B1	For $\tan^{-1}(2)$, or equivalent
		Enlargement with scale factor $\sqrt{5}$	B1	For stating 'enlargement'
			B1 **3**	For correct (exact) scale factor
	(iv)	$\begin{pmatrix}\frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}}\\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\end{pmatrix}$	M1	For correct $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$ pattern
			A1 **2**	For correct matrix in exact form
			11	

8	**(i)**	$\det \mathbf{M} = a(3-1) - 2(2-(-2)) - 1(-2-6)$ $= 2a$	M1 A1 **2**	For correct expansion process For showing given answer correctly
	(ii)	$\mathbf{M}^{-1} = \frac{1}{2a}\begin{pmatrix} 2 & -1 & 1 \\ -4 & a+2 & a-2 \\ -8 & a+4 & 3a-4 \end{pmatrix}$	M1 A1 B1 A1 **4**	For correct process for adjoint entries For at least 4 correct entries in adjoint For dividing by the determinant For completely correct inverse
	(iii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, with $a = 1$ So $x = 0, y = 1, z = 1$	B1 M1 A1 **3**	For correct statement involving inverse For carrying out the correct multiplication For all three correct values
	(iv)	Eliminating x gives $4y - 2z = 2$ So for consistency with 1st equn, $k = 1$	M1 M1 A1 **3**	For eliminating x from 2nd and 3rd equns For comparing two y-z equations For correct value for k
	(v)	Solving $x + 3y = 2, 3x - y = 0$ gives $x = \frac{1}{5}, y = \frac{3}{5}$ These values check in $2y - x = 1$, so soln exists	M1 A1 **2**	For using $x = z$ to solve a pair of equns For a completely correct demonstration
			14	