OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4726**

Further Pure Mathematics 2

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

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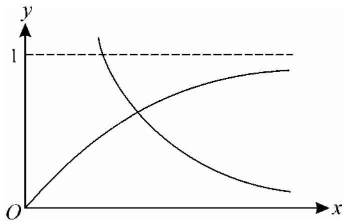
[Turn over

1	(i)	$\text{RHS} = 2\left(\frac{1}{2}(e^x + e^{-x})\right)^2 - 1 = \frac{1}{2}(e^{2x} + e^{-2x}) = \text{LHS}$	M1		For correct squaring of $(e^x + e^{-x})$
			A1	2	For completely correct proof
	(ii)	$2\cosh^2 x - 1 = k \Rightarrow \cosh x = \sqrt{\left(\frac{1}{2}(1+k)\right)}$	M1		For use of **(i)** and solving for $\cosh x$
			A1		For correct positive square root only
		$2\sinh^2 x + 1 = k \Rightarrow \sinh x = \pm\sqrt{\left(\frac{1}{2}(k-1)\right)}$	M1		For use of $\cosh^2 x - \sinh^2 x = 1$, or equivalent
			A1	4	For both correct square roots
				$\boxed{6}$	
2	(i)	$x = -1$ is an asymptote	B1		For correct equation of vertical asymptote
		$y = 2x + 1 + \dfrac{2}{x+1}$	M1		For algebraic division, or equivalent
		Hence $y = 2x + 1$ is an asymptote	A1	3	For correct equation of oblique asymptote
	(ii)	*EITHER*: Quadratic $2x^2 + (3-y)x + (3-y) = 0$	M1		For using discriminant of relevant quadratic
		has no real roots if $(3-y)^2 < 8(3-y)$	A1		For correct inequality or equation in y
		Hence $(3-y)(-5-y) < 0$	M1		For factorising, or equivalent
		So required values are 3 and -5	A1		For given answer correctly shown
		OR: $\dfrac{dy}{dx} = 2 - \dfrac{2}{(x+1)^2} = 0$	M1		For differentiating and equating to zero
		Hence $(x+1)^2 = 1$	A1		For correct simplified quadratic in x
		So $x = -2$ and $0 \Rightarrow y = -5$ and 3	M1		For solving for x and substituting to find y
			A1	4	For given answer correctly shown
				$\boxed{7}$	
3	(i)	*EITHER*: If $f(x) = \ln(x+2)$, then $f'(x) = \dfrac{1}{2+x}$	M1		For at least one differentiation attempt
		and $f''(x) = -\dfrac{1}{(2+x)^2}$	A1		For correct first and second derivatives
		$f(0) = \ln 2,\ f'(0) = \frac{1}{2},\ f''(0) = -\frac{1}{4}$	A1√		For all three evaluations correct
		Hence $\ln(x+2) = \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots$	A1		For three correct terms
		OR: $\ln(2+x) = \ln[2(1+\frac{1}{2}x)]$	M1		For factorising in this way
		$= \ln 2 + \ln(1+\frac{1}{2}x)$	A1		For using relevant log law correctly
		$= \ln 2 + \frac{1}{2}x - \dfrac{(\frac{1}{2}x)^2}{2} + \dots$	M1		For use of standard series expansion
		$= \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots$	A1	4	For three correct terms
	(ii)	$\ln(2-x) \approx \ln 2 - \frac{1}{2}x - \frac{1}{8}x^2$	B1√		For replacing x by $-x$
		$\ln\left(\dfrac{2+x}{2-x}\right) \approx (\ln 2 + \frac{1}{2}x - \frac{1}{8}x^2) - (\ln 2 - \frac{1}{2}x - \frac{1}{8}x^2)$	M1		For subtracting the two series
		$\approx x$, as required	A1	3	For showing given answer correctly
				$\boxed{7}$	

4	**(i)**	$r = 0 \Rightarrow \cos 2\theta = 0 \Rightarrow \theta = \pm\frac{1}{4}\pi, \pm\frac{3}{4}\pi$	M1 A1 A1 **3**	For equating r to zero and solving for θ For any two correct values For all four correct values and no others
	(ii)	Area is $\frac{1}{2}\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} 4\cos^2 2\theta \, \mathrm{d}\theta$ i.e. $\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} 1 + \cos 4\theta \, \mathrm{d}\theta = \left[\theta + \frac{1}{4}\sin 4\theta\right]_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} = \frac{1}{2}\pi$	M1 B1√ M1 A1 A1 **5**	For us of correct formula $\frac{1}{2}\int r^2 \, \mathrm{d}\theta$ For correct limits from **(i)** For using double-angle formula For $\theta + \frac{1}{4}\sin 4\theta$ correct For correct (exact) answer
			$\boxed{8}$	
5	**(i)**	LHS is the total area of the four rectangles RHS is the corresponding area under the curve, which is clearly greater	B1 B1 **2**	For identifying rectangle areas (not heights) For correct explanation
	(ii)	$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} > \int_0^4 \frac{1}{x+2} \, \mathrm{d}x$	M1 A1 **2**	For attempt at relevant new inequality For correct statement
	(iii)	Sum is the area of 999 rectangles Bounds are $\int_0^{999} \frac{1}{x+2} \, \mathrm{d}x$ and $\int_0^{999} \frac{1}{x+1} \, \mathrm{d}x$ So lower bound is $\left[\ln(x+2)\right]_0^{999} = \ln(500.5)$ and upper bound is $\left[\ln(x+1)\right]_0^{999} = \ln(1000)$	M1 M1 A1 A1 **4**	For considering the sum as an area again For stating either integral as a bound For showing the given value correctly Ditto
			$\boxed{8}$	
6	**(i)**	$I_n = \left[-\frac{2}{3}x^n(1-x)^{\frac{3}{2}}\right]_0^1 + \frac{2}{3}n\int_0^1 x^{n-1}(1-x)^{\frac{3}{2}} \, \mathrm{d}x$ $= \frac{2}{3}n\int_0^1 x^{n-1}(1-x)\sqrt{(1-x)} \, \mathrm{d}x$ $= \frac{2}{3}n(I_{n-1} - I_n)$ Hence $(2n+3)I_n = 2nI_{n-1}$, as required	M1 A1 M1 M1 A1 A1 **6**	For using integration by parts For correct first stage result For use of limits in integrated term For splitting the remaining integral up For correct relation between I_n and I_{n-1} For showing given answer correctly
	(ii)	$I_2 = \frac{4}{7}I_1 = \frac{4}{7} \times \frac{2}{5}I_0$ Hence $I_2 = \frac{8}{35}\left[-\frac{2}{3}(1-x)^{\frac{3}{2}}\right]_0^1 = \frac{16}{105}$	M1 A1 M1 A1 **4**	For two uses of the recurrence relation For correct expression in terms of I_0 For evaluation of I_0 For correct answer
			$\boxed{10}$	

Q	Part	Answer	Mark	Total	Guidance
7	(i)	$\dfrac{dy}{dx}=\dfrac{\cosh x - x\sinh x}{\cosh^2 x}$	M1		For differentiating and equating to zero
		Max occurs when $\cosh x = x\sinh x$, i.e. $x\tanh x = 1$	A1	2	For showing given result correctly
	(ii)		B1		For correct sketch of $y=\tanh x$
			B1		For identification of asymptote $y=1$
			B1	3	For correct explanation of $\alpha>1$ based on intersection $(1,1)$ of $y=1/x$ with $y=1$
	(iii)	$x_{n+1}=x_n-\dfrac{x_n\tanh x_n - 1}{\tanh x_n + x_n \operatorname{sech}^2 x_n}$	M1		For correct Newton-Raphson structure
			A1		For all details in $x-\dfrac{f(x)}{f'(x)}$ correct
		$x_1=1 \Rightarrow x_2 = 1.20177...$	M1		For using Newton-Raphson at least once
			A1		For x_2 correct to at least 3sf
		$x_3=1.1996785...$	A1	5	For x_3 correct to at least 4sf
	(iv)	$e_1\approx 0.2,\ e_2\approx -0.002$	B1√		For both magnitudes correct
		$\dfrac{e_3}{e_2^2}\approx\dfrac{e_2}{e_1^2}\Rightarrow e_3\approx -2\times10^{-7}$	M1		For use of quadratic convergence property
			A1	3	For answer of correct magnitude
				$\boxed{13}$	
8	(i)	$\dfrac{dt}{dx}=\tfrac{1}{2}(1+t^2)$	B1		For this relation, stated or used
		$\displaystyle\int_0^{\frac{1}{2}\pi}\sqrt{\frac{1-\cos}{1+\sin x}}\,dx=\int_0^1\sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}}.\frac{2}{1+t^2}\,dt$	M1		For complete substitution for x in integrand
			B1		For justification of limits 0 and 1 for t
		$\displaystyle=\int_0^1\sqrt{\frac{2t^2}{(1+t)^2}}.\frac{2}{1+t^2}\,dt=2\sqrt{2}\int_0^1\frac{t}{(1+t)(1+t^2)}\,dt$	A1	4	For correct simplification to given answer
	(ii)	$\dfrac{t}{(1+t)(1+t^2)}=\dfrac{A}{1+t}+\dfrac{Bt+C}{1+t^2}$	B1		For statement of correct form of pfs
		Hence $t\equiv A(1+t^2)+(Bt+C)(1+t)$	M1		For any use of the identity involving B or C
		From which $A=-\tfrac{1}{2},\ B=\tfrac{1}{2},\ C=\tfrac{1}{2}$	B1		For correct value of A
			A1		For correct value of B
			A1	5	For correct value of C
	(iii)	Int is $2\sqrt{2}\left[-\tfrac{1}{2}\ln(1+t)+\tfrac{1}{4}\ln(1+t^2)+\tfrac{1}{2}\tan^{-1}t\right]_0^1$	B1√		For both logarithm terms correct
			B1√		For the inverse tan term correct
		$=\tfrac{1}{4}(\pi-2\ln 2)\sqrt{2}$	M1		For use of appropriate limits
			A1	4	For correct (exact) answer in any form
				$\boxed{13}$	