

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4730

Mechanics 3

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72
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This mark scheme consists of 4 printed pages.

<p>1 $0.2 = \frac{2\pi}{\omega} \Rightarrow \omega = 10\pi$</p> <p>Hence maximum speed is $0.3 \times 10\pi = 3\pi \approx 9.42 \text{ m s}^{-1}$</p> <p>Maximum acc is $0.3 \times (10\pi)^2 = 30\pi^2 \approx 296 \text{ m s}^{-2}$</p>	<p>M1 A1 M1 A1✓ M1 A1✓</p> <p>6 6</p>	<p>For relevant use of $\frac{2\pi}{\omega}$</p> <p>For correct value 10π</p> <p>For relevant use of $v = a\omega$</p> <p>For correct value 3π or 9.42</p> <p>For relevant use of $a\omega^2$</p> <p>For correct value 30π or 296</p>
<p>3 (i) A and B move off \perp and \parallel resp. to line of centres</p> <p>$2mv_B = mu \cos \theta$</p> <p>$v_B = eu \cos \theta$</p> <p>Hence $e = 0.5$</p> <hr/> <p>(ii) $v_A = u \sin \theta$</p> <p>Hence $v_A = v_B \Rightarrow u \sin \theta = 0.5u \cos \theta$</p> <p>So $\theta = \tan^{-1} 0.5 \approx 26.6^\circ$</p>	<p>M1 A1 A1 A1</p> <p>4</p> <hr/> <p>B1 M1 A1</p> <p>3 7</p>	<p>For correct directions of motion after impact</p> <p>For correct momentum equation</p> <p>For correct restitution equation</p> <p>For correct answer 0.5</p> <hr/> <p>For correct equation</p> <p>For forming the relevant equation for θ</p> <p>For correct value 26.6</p>
<p>3 $80\,000v \frac{dv}{dx} = -(27\,000 + 50v^2)$</p> <p>Hence $x = -\int \frac{1600v}{540 + v^2} dv$</p> <p>$= -800 \ln(540 + v^2) + k$</p> <p>$v = 90$ when $x = 0 \Rightarrow k = 800 \ln 8640$</p> <p>Hence when $v = 0$, $x = 800 \ln 16$</p> <p>So distance is 2220 m approximately</p>	<p>M1 A1 M1 M1 A1✓ M1 M1 A1</p> <p>8 8</p>	<p>For using Newton II to form a DE</p> <p>For correct equation including $v \frac{dv}{dx}$</p> <p>For separation of variables</p> <p>For logarithmic form of integral</p> <p>For correct integration of (their) $\frac{av}{b + cv^2}$</p> <p>For use of initial condition to find k</p> <p>For evaluation of required distance</p> <p>(The previous two M marks can equivalently be earned by using definite integration)</p> <p>For correct value 2220</p>
<p>4 (i) Greatest tension $= \frac{1320 \times 35}{25} = 1848 \text{ N}$</p> <hr/> <p>(ii) (a) $mg \times 60 = \frac{1320}{2 \times 25} (60 - 25)^2$</p> <p>Hence the girl's mass is 55 kg</p> <hr/> <p>(b) $55g \times 30 = \frac{1}{2} \times 55v^2 + \frac{1320}{2 \times 25} \times (30 - 25)^2$</p> <p>So $v^2 = 564$, hence speed is 23.7 m s^{-1}</p>	<p>M1 A1</p> <p>2</p> <hr/> <p>M1 A1 M1 A1</p> <p>4</p> <hr/> <p>M1 A1✓ A1</p> <p>3 9</p>	<p>For use of $\frac{\lambda x}{l}$ at lowest point</p> <p>For correct answer 1848</p> <hr/> <p>For use of correct EPE formula $\frac{\lambda x^2}{2l}$</p> <p>For correct unsimplified expression for EPE</p> <p>For use of equation involving EPE and GPE</p> <p>For correct answer 55</p> <hr/> <p>For energy equation with KE, GPE and EPE</p> <p>For equation with all terms correct</p> <p>For correct answer 24.3</p>

5	(i) Radial acc is $\frac{6.5^2}{1.5} = 28.2 \text{ m s}^{-2}$ Transverse acc is $g \sin 40^\circ = 6.30 \text{ m s}^{-2}$	B1 B1	2	For correct value 28.2 For correct value 6.30
	(ii) $\frac{1}{2} \times 0.3 \times (6.5^2 - v^2) = 0.3 \times 9.8 \times 1.5 (\cos 40^\circ - \cos \theta^\circ)$ Hence $42.25 - v^2 = 29.4 (\cos 40^\circ - \cos \theta^\circ)$ i.e. $v^2 \approx 19.7 + 29.4 \cos \theta^\circ$	M1 B1 B1 A1	4	For equating PE gain to KE loss, or equiv For correct expression for PE gain For correct expression for KE loss For showing given answer correctly
	(iii) $T - 0.3g \cos \theta^\circ = 0.3 \times \frac{v^2}{1.5}$ Hence $T = 2.94 \cos \theta^\circ + 0.2(19.7 + 29.4 \cos \theta^\circ)$ $= 3.95 + 8.82 \cos \theta^\circ$	M1 A1 M1 A1	4	For use of Newton II, including $\frac{mv^2}{r}$ term For correct (unsimplified) equation For substitution, to obtain expression for T For correct answer
	(iv) $T = 0$ when $3.95 + 8.82 \cos \theta^\circ = 0$ Hence $v^2 = 19.7 + 29.4 \times \left(-\frac{3.95}{8.82}\right) \Rightarrow v \approx 2.56$	M1 M1 A1	3	For equating T to zero to find $\cos \theta$ For using this $\cos \theta$ to find v For correct answer 2.56
13				
6	(i) Mom @ B for BAC: $V_C \times 1.6 = 150 \times 0.4 + 270 \times 1.2$ Hence $V_C = 240$ Mom @ C for AC: $V_A \times 0.8 + H_A \times 2.5 = 270 \times 0.4$ Res \uparrow for AC: $V_A + V_C = 270$ Hence $V_A = 270 - 240 = 30 \text{ N}$ (upwards) and $2.5H_A = 108 - 0.8 \times 30 \Rightarrow H_A = 33.6 \text{ N}$ (right)	M1 A1 M1 A1 M1 A1	8	For suitable moments equation for BAC For correct value for V_C (or equivalent) For a moments equation for one rod with all required forces included For a correct equation For another equation leading to V_A For correct magnitude and direction For substituting back to find H_A For correct magnitude and direction
	(ii) $V_B = 270 + 150 - V_C = 180$ $H_B = H_C = H_A = 33.6$ $\frac{H_B}{V_B} = \frac{33.6}{180} = 0.187, \frac{H_C}{V_C} = \frac{33.6}{240} = 0.14$ Hence friction is limiting at B Value of μ is 0.187	M1 A1✓ A1✓ M1 A1✓ A1✓	6	For finding all of V_B, H_B and H_C For correct V_B For both H_B and H_C correct For considering ratios at B and C, or equiv For identifying point with larger ratio For identifying the larger ratio as μ
14				

<p>7 (i) $T_{AP} = \frac{196}{0.8} \times (1.5 - 0.8) = 171.5$ $T_{BP} = \frac{196}{0.8} \times (2.6 - 1.5 - 0.8) = 73.5$ $T_{AP} - T_{BP} = 98 = 10g$, hence equilibrium</p>	<p>M1 A1 M1 A1</p>	<p>For using Hook's law to find either tension For both tensions correct For considering $T_{AP} = mg + T_{BP}$, or equiv 4 For showing given result correctly</p>
<p>(ii) Extension of PA is $1.5 + x - 0.8 = 0.7 + x$ Hence $T_{AP} = \frac{196}{0.8}(0.7 + x) = 245(0.7 + x)$ and $T_{BP} = \frac{196}{0.8}(1.1 - x - 0.8) = 245(0.3 - x)$</p>	<p>M1 A1 A1</p>	<p>For finding either extension in terms of x For showing one given answer correctly 3 For showing the other given answer correctly</p>
<p>(iii) $245(0.3 - x) + 10g - 245(0.7 + x) = 10\ddot{x}$ Hence $\ddot{x} = -49x$, so the motion is SHM</p>	<p>M1 A1 A1</p>	<p>For use of Newton II, at a general position For a correct equation 3 For showing the given result correctly</p>
<p>(iv) $0.2 = 0.25\cos(7t)$ Hence half of time above mid-pt is $t = 0.0919\dots$ Proportion is $\frac{t}{\pi/\omega} = 0.205$</p>	<p>M1 A1 A1 M1 A1</p>	<p>For use of ± 0.2 in SHM equation involving t For a correct equation for a relevant time For correct value for a relevant time For relating t to period of oscillation 5 For correct proportion 0.205</p>
		<div style="border: 1px solid black; padding: 2px; display: inline-block;">15</div>