OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4733

Probability & Statistics 2

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

Registered Charity Number: 1066969
[Turn over

1	**(i)**	$\frac{12.0}{\sqrt{n}} = 1.50 \Rightarrow \sqrt{n} = \frac{12.0}{1.50} = 8 \Rightarrow n = 64$	B1 M1 A1	**3**	For any correct equation involving n For correct solution method for n or $\sqrt{n}$ For correct answer 64
	(ii)	n is large, the distribution of $\bar{F}$ can be taken to be normal, according to the Central Limit Theorem	M1 A1	**2**	For relating the size of n to normality For reference to the CLT
				5	
2	**(i)**	Reasons for bias may include: Larger properties more likely to be picked Some regions of the map more/less likely	 B1 B1	 **2**	 For stating one valid relevant reason For stating a second valid relevant reason
	(ii)	Make a list of all the houses in the neighbourhood Number the houses from 1 upwards Select the sample using random numbers	B1 B1 B1	**3**	For stating or implying a sampling frame For numbering the sampling units For referring to use of random numbers
				5	
3	**(i)**	$\frac{1}{36}$	B1	**1**	For correct probability
	(ii)	Number obtaining two sixes $\sim$ B(60, $\frac{1}{36}$) Approximate distribution is Po($\frac{5}{3}$) $P(\geqslant 4) = 1 - e^{-\frac{5}{3}}\left\{1 + \frac{5}{3} + \frac{(5/3)^2}{2!} + \frac{(5/3)^3}{3!}\right\}$ $= 0.0883$	M1 A1√ M1 M1 A1	**5**	For stating or implying binomial distribution For the correct Poisson approximation For calculation of correct terms For correct use of Poisson formula For correct answer 0.088(3)
				6	
4	**(i)**	**(a)** $\frac{15.0 - 20.0}{\sigma} = -0.253$ Hence $\sigma = \frac{5}{0.253} \approx 19.8$	M1 B1 M1 A1	**4**	For standardising and equating to $\Phi^{-1}(p)$ For correct value 0.253 (or 0.254) seen For solving equation for σ For correct value 19.8
		(b) $g = 25.0$, using symmetry Hence $P(G > 2g) = 1 - \Phi\left(\frac{50.0 - 20.0}{19.8}\right)$ $= 1 - 0.935 = 0.065$	B1 M1 A1	**3**	For stating (or finding) the value of g For correct process for upper tail prob For correct answer
	(ii)	If normal, $P(G < 0)$ is substantial Hence the assumption seems unjustified	M1 A1	**2**	For considering relevant normal probability For stating the appropriate conclusion
				9	

5	$\bar{x} = \frac{4070}{50} = 81.4$	B1	For correct value of sample mean
	$s^2 = \frac{336\,100}{49} - \frac{4070^2}{49 \times 50} = 98$	M1	For calculation of unbiased or biased estimate
		A1	For correct value of unbiased estimate
	$H_0 : \mu = 84.0; \quad H_1 : \mu < 84.0$	B1	For correct statement of null hypothesis
		B1	For correct statement of alt hypothesis
	EITHER: $z = \frac{\bar{x} - 84.0}{\sqrt{(s^2/50)}} = -1.857$	M1	For standardising, including use of $\sqrt{50}$
		A1	For correct value 1.857
	This is significant, since $-1.857 < -1.645$	M1	For comparing z value to -1.645 or equiv
	OR: $\frac{c - 84.0}{\sqrt{(s^2/50)}} = -1.645 \Rightarrow c = 81.697$	M1	For critical value calculation, inc use of $\sqrt{50}$
		A1	For correct value 81.697
	$\bar{x}$ is in the critical region since $81.4 < 81.697$	M1	For comparing sample mean to critical region
	Hence H_0 is rejected	A1√	For stating or implying rejection of H_0
	There is sufficient evidence to conclude that the mean solubility rating is less than 84.0	A1√ **10**	For stating the outcome in context
		10	
6	**(i)** **(a)** For one day, the distribution is Po(0.5)	B1	For use of correct Poisson mean
	Hence P(exactly 2) = 0.9856 – 0.9098	M1	For relevant use of tables (or formula)
	= 0.0758	A1 **3**	For correct answer 0.0758
	(b) No. of days with no cars ~ B(365, 0.6065)	M1	For relevant Poisson probability of P(0)
		A1	For identifying correct binomial distribution
	Normal approximation is N(221.3725, 87.11)	A1√	For correct use of *np* and *npq*
	$P(< 205) = P\left(Z < \frac{204.5 - 221.3725}{\sqrt{87.11}}\right)$	M1	For standardising (with or without c.c. here)
		A1	For completely correct expression
	$= \Phi(-1.808) = 0.0353$	A1 **6**	For correct answer 0.0353
	(ii) Events (cars running out of petrol) must occur at a constant average rate. This seems unlikely, given that there will be different volumes of traffic on different days of the week (e.g. weekdays and weekends)	B1	For correct statement of the condition
		B1 **2**	For a correct explanation
		11	

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7	**(i)**	$1 = k\int_0^3 (9x - x^3)\,\mathrm{d}x = k\left[\frac{9}{2}x^2 - \frac{1}{4}x^4\right]_0^3 = \frac{81}{4}k$	M1		For equating to 1 and integrating
		Hence $k = \frac{4}{81}$	A1	**2**	For showing given answer correctly
	(ii)	$\mathrm{E}(X) = \frac{4}{81}\int_0^3 x^2(9 - x^2)\,\mathrm{d}x = \frac{4}{81}\left[3x^3 - \frac{1}{5}x^5\right]_0^3 = 1.6$	M1		For attempt at $\int_0^3 x\,\mathrm{f}(x)\,\mathrm{d}x$
			A1		For correct indefinite integral, in any form
			A1	**3**	For correct answer 1.6
	(iii)	**(a)** $\frac{3}{5} = \frac{4}{81}\int_0^y x(9 - x^2)\,\mathrm{d}x = \frac{4}{81}\left[\frac{9}{2}x^2 - \frac{1}{4}x^4\right]_0^y$	M1		For attempt at $\int_0^y \mathrm{f}(x)\,\mathrm{d}x = \frac{3}{5}$
			B1		For correct indefinite integral, in any form
		Hence $\frac{3}{5} = \frac{4}{81}\left\{\frac{9}{2}y^2 - \frac{1}{4}y^4\right\}$	M1		Use limits to produce relevant equation in y
		i.e. $5y^4 - 90y^2 + 243 = 0$	A1	**4**	For showing given answer correctly
		(b) $w = \dfrac{90 \pm \sqrt{(90^2 - 4\times5\times243)}}{10} = 3.31$ or 14.7	M1		For use of quadratic formula to find w
			A1		For either value found correctly
		Hence $y = \sqrt{3.31} = 1.82$	A1	**3**	For correct (unique) answer 1.82
				12	
8	**(i)**	$\mathrm{H}_0 : p = 0.15$; $\mathrm{H}_1 : p > 0.15$	B1		For correct statement of null hypothesis
			B1		For correct statement of alt hypothesis
		Under H_0, number left-handed $L \sim \mathrm{B}(12, 0.15)$	M1		For correct distribution stated or implied
		$\mathrm{P}(L \geqslant 5) = 1 - 0.9761 = 0.0239$	M1		For calculation of relevant tail probability, or finding the critical region
			A1		For correct value 0.0239 or region $l \geqslant 5$
		This is significant, since $0.0239 < 0.05$	M1		For comparing tail probability with 0.05 or observed value with critical region
		Hence H_0 is rejected	A1√		For stating or implying rejection of H_0
		Accept the suggestion that the proportion of mathematicians who are left-handed is more than 15%	A1√	**8**	For stating the outcome in context
	(ii)	$\mathrm{P_I} = \mathrm{P}(L$ in critical region$) = 0.0239$	M1		For evaluating $\mathrm{P}(\text{reject } \mathrm{H}_0)$
			A1	**2**	For correct answer 0.0239 or equivalent
	(iii)	$\mathrm{P_{II}} = \mathrm{P}(L \leqslant 4 \mid p = 0.2) = 0.9274$	M1		For evaluating $\mathrm{P}(\text{accept } \mathrm{H}_0)$ with $p = 0.2$
			A1	**2**	For correct probability
	(iv)	$\mathrm{P_{II}} = 0.0188$ for $p = \frac{2}{3}$ and 0.0095 for $p = 0.7$	M1		For relevant use of tables
		So the proportion is between 67% and 70%	A1	**2**	For an appropriate conclusion
				14	