OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4727

Further Pure Mathematics 3

Specimen Paper

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 Find the general solution of the differential equation

$$\frac{dy}{dx} - \frac{y}{x} = x,$$

giving y in terms of x in your answer. [5]

2 The set $S = \{a, b, c, d\}$ under the binary operation $*$ forms a group G of order 4 with the following operation table.

$*$	a	b	c	d
a	d	a	b	c
b	a	b	c	d
c	b	c	d	a
d	c	d	a	b

(i) Find the order of each element of G. [3]

(ii) Write down a proper subgroup of G. [1]

(iii) Is the group G cyclic? Give a reason for your answer. [1]

(iv) State suitable values for each of a, b, c and d in the case where the operation $*$ is multiplication of complex numbers. [1]

3 The planes Π_1 and Π_2 have equations $\mathbf{r}.(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) = 1$ and $\mathbf{r}.(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 3$ respectively. Find

(i) the acute angle between Π_1 and Π_2, correct to the nearest degree, [4]

(ii) the equation of the line of intersection of Π_1 and Π_2, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [4]

4 In this question, give your answers exactly in polar form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

(i) Express $4((\sqrt{3}) - \mathrm{i})$ in polar form. [2]

(ii) Find the cube roots of $4((\sqrt{3}) - \mathrm{i})$ in polar form. [4]

(iii) Sketch an Argand diagram showing the positions of the cube roots found in part **(ii)**. Hence, or otherwise, prove that the sum of these cube roots is zero. [3]

5 The lines l_1 and l_2 have equations

$$\frac{x-5}{1} = \frac{y-1}{-1} = \frac{z-5}{-2} \quad \text{and} \quad \frac{x-1}{-4} = \frac{y-11}{-14} = \frac{z-2}{2}.$$

(i) Find the exact value of the shortest distance between l_1 and l_2. [5]

(ii) Find an equation for the plane containing l_1 and parallel to l_2 in the form $ax + by + cz = d$. [4]

6 The set S consists of all non-singular 2×2 real matrices $\mathbf{A}$ such that $\mathbf{AQ} = \mathbf{QA}$, where

$$\mathbf{Q} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

(i) Prove that each matrix $\mathbf{A}$ must be of the form $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$. [4]

(ii) State clearly the restriction on the value of a such that $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$ is in S. [1]

(iii) Prove that S is a group under the operation of matrix multiplication. (You may assume that matrix multiplication is associative.) [5]

7 **(i)** Prove that if $z = e^{i\theta}$, then $z^n + \dfrac{1}{z^n} = 2\cos n\theta$. [2]

(ii) Express $\cos^6\theta$ in terms of cosines of multiples of θ, and hence find the exact value of

$$\int_0^{\frac{1}{3}\pi} \cos^6\theta \, d\theta.$$ [8]

8 **(i)** Find the value of the constant k such that $y = kx^2 e^{-2x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 2e^{-2x}.$$ [4]

(ii) Find the solution of this differential equation for which $y = 1$ and $\dfrac{dy}{dx} = 0$ when $x = 0$. [7]

(iii) Use the differential equation to determine the value of $\dfrac{d^2y}{dx^2}$ when $x = 0$. Hence prove that $0 < y \leqslant 1$ for $x \geqslant 0$. [4]